DNS OF A HOT-WIRE ANEMOMETER IN A TURBULENT CHANNEL FLOW.

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Abstract

A body with a shape similar to a hot wire with its sheath, but no prongs, has been placed close to the wall of a turbulent channel at $Re_\tau = 600$. The results of the channel flow, without the wire, agree with previous published ones, despite the modest resolution and domain size. A simplified, two-dimensional version of the wire at the same Reynolds number has been studied to compare the dynamic response of *cold* and *hot* wires, where a slightly bigger perturbation is seen in the *hot* case, but an almost identical dynamic response. The *cold* wire seems to be able to measure instantaneous velocity with total drag after proper calibration. Being a DNS, the complete description of the flow field around the wire is obtained.

1 Introduction

This work started as an interest-only project, its spirit is exploratory, and it is open for collaboration. It has two equally important goals. The first one is to explore the limits of a general computational fluid dynamics solver like OpenFOAM[1] for Direct Numerical Simulations (DNS) of turbulent flows in moderately complex geometrical configurations. The second one is to extend the present knowledge and the implications of using hot-wire anemometry to measure the properties of wall bounded turbulent flows.

There are a handful of references about DNS of turbulent flows with OpenFOAM, like Komen et al. (2014) for pipes and channels, and Vuorinen et al. (2014) for channels and mixing layers. These two studies conclude that OpenFOAM, like other CFD solvers, can produce correct DNS of canonical configurations. Instead of checking the correctness of the solution with a canonical case, the goal of this project is to obtain a solution where a specific purpose solver cannot be used. The choice has been to simulate a hot-wire anemometer within a channel flow, at the point where the turbulent intensity is highest, precisely where the validity of the hypotheses that support this measurement technique is less clear. This case has a relatively complex geometry, and it is interesting for the experimental study of wall-bounded turbulent flows.

Hot-wire anemometry is still one of the most successful experimental techniques to measure velocity components in turbulent flows. Hot wires are cheap, relatively simple to set up, and have excellent spatial and temporal resolution. Despite hot-wire anemometry being well established nowadays, and that many details can be found in handbooks like Tropea et al. (2007), its limitations are still under discussion. Three of them are considered in this study.

The first one is the validity of Taylor's *frozen turbulence* hypothesis, the assumption that the advection of a turbulent field passed a fixed point is due entirely to the mean flow. This hypothesis was proven to break down in the case of flows with strong shear by Lin (1953), where turbulent intensities u' are comparable to the magnitude of the mean flow U. This assumption may lead to wrong interpretations when wall bounded flows are measured, since the relation between temporal and spatial frequency is not necessarily constant, and equal to the average component of the streamwise velocity. This effect is described in del Álamo and Jiménez (2009), quantified as a function of the wavenumber and the Reynolds number, and a method to correct this effect is given too. A more explanatory summary of the latter article can be found in Moin (2009).

The second one is related to the effect of the hot wire length. Hot wires have a slender shape, with a characteristic diameter many times smaller than its length. When the length of the wire is larger than the smallest scales found in the fluid motion, the anemometer acts approximately as a filter. This behavior is a limiting factor when measuring high Reynolds number wall-bounded turbulent flows, when the Kolmogorov length η is smaller that the size of the anemometer. This is a first order error, given that the generated signal is not able to capture all the energy-containing scales in the flow, and the turbulent intensities are therefore underestimated. This effect was investigated in Ligrani and Bradshaw (1987) for the

[1] OpenFOAM is a registered trademark of SGI corp.

case of single hot wires, and extended in Sillero and Jiménez (2013) to include cross wires.

The third limitation, also addressed in Sillero and Jiménez (2013), has to do with what the wire is actually measuring. In the case of a single wire aligned with a given coordinate, the measurement is a combination of the other two components. In the case of wall-bounded flows, if U, V, and W are the streamwise, wall-normal, and spanwise velocities respectively, a wire aligned with the W component will measure a combination of U and V. This caveat can be solved by using a combination of wires, but at the cost of a larger perturbation on the fluid.

The assessment of the measurement errors introduced by hot-wire anemometers in previous works is done *a posteriori*, based on how direct numerical simulation data has to be modified to fit anemometer measurements. This allows the estimation of each of the previously mentioned errors one at a time, but since the behavior of a hot-wire anemometer is probably not linear, superposition does not necessarily apply. If it is possible to simulate a probe in a realistic environment such as a channel flow, it will provide an *a priori* approximation of the flow field without any additional modeling other than numerics, and will help to understand how the mentioned effects are combined. This is, however, a complex simulation that has never been successfully completed to the knowledge of the authors. This work is an attempt to produce such simulation with OpenFOAM, and to determine its viability, cost, and correctness.

2 Simulation of a turbulent channel flow

A DNS of a channel flow is carried out at $Re_\tau \simeq 600$, where the Reynolds number is defined as $Re_\tau = u_\tau h/\nu$, with u_τ the friction velocity, h the channel half-height, and ν the kinematic viscosity. The extent of the domain in the wall-normal, streamwise and spanwise direction is $2h \times \pi h \times \pi h/2$. The discretization is a structured rectangular-prism mesh based on a $129 \times 192 \times 192$ array of nodes, that will become unstructured as soon as the wire is placed within the domain.

A turbulent channel has been chosen because it is a flow with a strong shear close to the wall, can be simulated with a reasonable computational cost, and is a realistic wall bounded flow. It is also well understood, and many high quality simulations in the same range of Reynolds numbers are available to compare with. On the other hand, this channel is a periodic flow in both streamwise and spanwise directions, and any object placed in it will become an infinite array of identical objects with the same periodicity.

This simulation is not straightforward in OpenFOAM. The pressure gradient along the channel had to be enforced by a momentum source computed at each time step. The average flow-rate is calculated and compared with the desired value. From this, the required pressure gradient to obtain the desired average velocity is computed.

A comparison of the present simulation with some previous cases at similar Reynolds numbers, after running approximately $10h/u_\tau$, is shown in figure 1.

(a)

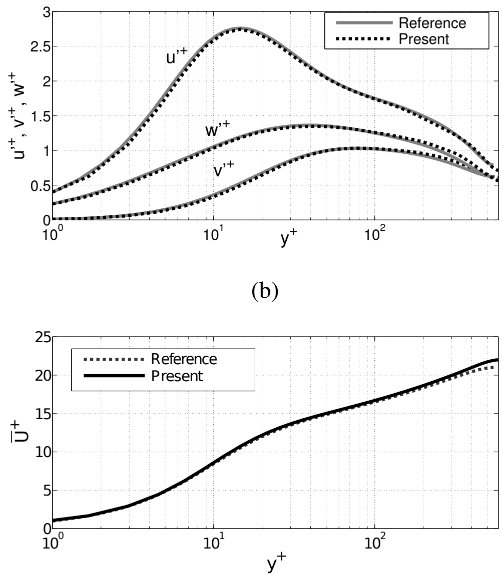

Figure 1: Comparison of the present turbulent channel simulation with del Álamo and Jiménez (2003): (a) turbulent intensities, and (b) mean velocity profile.

The difference between the present result and the reference simulation is more important in the center of the channel, caused by the fact that large scales take longer to converge. This is a known effect due to the size of the domain, since channel flows are affected by the domain size, and the intensity of the wake in the center of the channel is inversely proportional to the aspect ratio of the domain during the transient. This effect is described and quantified in Flores and Jiménez (2010).

This discrepancy does not mean that the complete simulation is flawed, since the dynamics of the turbulent structures in the buffer and logarithmic layers can be correct regardless of the differences in the center of the channel. The size of the domain is still large enough to contain turbulent structures in their “natural” state close to the wall, precisely where the wire is placed. The consequence of using a smaller domain is that a higher Reynolds number is achieved with a similar amount of degrees of freedom to Komen et al. (2014), and Vuorinen et al. (2014). A good guide on how to simulate channel flows with small domains can be found in Lozano-Durán and Jiménez (2014).

3 Velocity measurements based on total drag.

Hot-wire anemometers also suffer from errors due to thermal effects, like forced convection and radiation. Those effects can be important, and should not be neglected in a proper experimental setup. In this case, however, they will not be taken into account. The reason is that with the expected temperature gradients, a fully compressible solver is needed, and since the Prandtl number of the usual working fluids is smaller than one, the mesh should be finer than the case without thermal effects. One objective of this work is to keep the numerical setup as computationally cheap as possible. For this reason, the case will be solved with the incompressible solver pimpleFoam (OpenFOAM (2013)), and the errors present due to this naive modeling exercise will be approximated.

Instead of using the total heat transfer of the wire, the drag is used. At such low Reynolds number (the wire works at a Reynolds number based on its diameter always smaller than unity), there is a law of proportionality between drag and velocity (White (1946), and Tritton (1959)), similar to King's law (King (1919)). Drag can be used to infer velocity, is cheaper to compute, and can be calibrated in a similar way to an actual hot-wire anemometer.

Two-dimensional wire, and thermal effects.

One question that now arises is if the flow surrounding the hot-wire anemometer is significantly different in the case with and without thermal effects or, in other words, if a *cold* wire in an incompressible flow behaves similarly to a *hot* one in a compressible flow. The following two-dimensional numerical experiment has been designed to find an answer. An infinite wire has been placed in the center of a very large domain, with a far field component of velocity of $U_\infty = 1m/s$, and a far field temperature $T_\infty = 300K$. The fluid's viscosity and density corresponds to the properties of air, and the diameter is defined to provide a Reynolds number based on the circle's diameter of $Re_D = 0.85$. Two different simulations are performed, one without temperature difference between the circle and the far field, $T_w = T_\infty = 300K$, and another one with $T_w = 500K$. Both simulations have been completed with a compressible solver, rhoPimpleFoam (OpenFOAM (2013)). The temperature at the surface of the simplified wire is kept constant, and tries to model Constant Temperature Anemometry (CTA). The Re_D of this simplified case is similar to the large wire simulated in section 4, and the temperature difference is comparable to an actual hot-wire anemometer.

The result of the simulation is presented in figure 2, where it can be seen that the flow field close to the wire is very similar in both cases. The perturbation caused by the presence of the wire is larger when a temperature difference exists, but the shape is very similar. This suggests that, although the case without thermal effects not being identical, the error is moderate, and it can be used to evaluate how the presence of the wire perturbs the surrounding fluid.

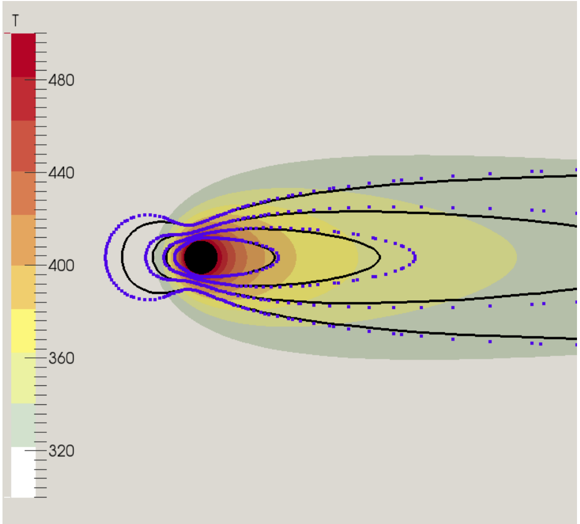

Figure 2: Contour levels of streamwise velocity for the simplified wire, at 0.2, 0.4, 0.6, and 0.8 U_∞. The solid black lines correspond to the *cold* wire, while the dotted blue lines correspond to a *hot* wire at $T_w = 500K$. The image in the background is a map of the temperature distribution, where the unshaded area corresponds to approximately unheated flow, at a temperature of $300K$.

Another important question is if the dynamic response of drag is as fast as the heat lost by the wire at a given time. One of the reasons why hot-wire anemometers are used to measure wall-bounded turbulent flows is their excellent temporal response. The previous experiment is modified to roughly follow Khoo et al. (1998), where a hot-wire anemometer is put in an environment with a known far-field temporal perturbation. In this case, the far-field is uniform in space, but changes in time following a flat spectrum with a cutoff frequency. This frequency corresponds to a wavelength of $\lambda_x = 30^+$ after the Taylor's analogy has been applied, which models the streamwise length of the smallest energy-containing eddies in a channel flow. This far-field perturbation can be considered to be slow, with a Strouhal number based on the circle's diameter of $St_D = 10^{-2}$. This is two orders of magnitude smaller than Re_D.

The results are presented in figure 3. The heat flux per unit length $\dot{q}$ is one term of King's law, and can be used to determine the dynamic behavior of a hot-wire anemometer. The same experiment is repeated without thermal effects, and the total drag per unit length is obtained for the *cold* wire. The three signals have been rescaled to contain the same amount of energy. It is shown that heat flux and drag have a very similar dynamic response for any frequency lower than the cutoff. This verifies that there exists a mapping between velocity and both measures in the given range of Reynolds and Strouhal number, and that the hot-wire anemometer is able to capture the characteristic frequencies of the phenomenon. Therefore, we conclude that the heat flux and the total drag can be used to measure velocity after proper calibration. This is equivalent to consider that the Reynolds analogy can be applied to this configuration.

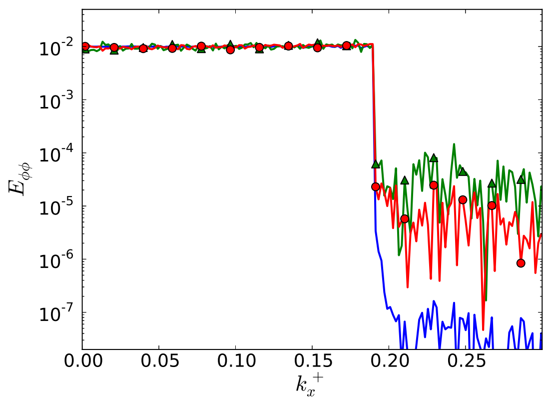

Figure 3: Energy spectrum of the signals obtained from the dynamic response of the infinite wire: (— blue) E_{uu}, (△ green) heat flux per unit length $E_{\dot{q}\dot{q}}$, (○ red) drag force per unit length E_{dd}.

4 *Cold* wire in a channel flow

An isolated hot-wire anemometer with infinite length has been studied in the previous section, confirming that the thermal effects represent a small correction to the incompressible solution. In the following experiment, a body with a shape similar to a hot wire with its sheath, but no prongs, has been placed close to the wall of the turbulent channel flow defined in section 2. The goal of this experiment is to explore the interaction between the channel flow and the probe, and to understand how the measure obtained by a hot-wire anemometer is influenced by the probe itself.

Given a channel flow, one can put to test any hot-wire anemometer in any position and condition. Since the goal of this project is to understand the possible limitations of this experimental technique, it is reasonable to place it where the conditions are most adverse, such as where the relative turbulent intensity u'/U is maximum. This location is at a distance of 15^+ from the wall (see Kim, Moin and Moser (1987)), where the $+$ superindex indicates the use of wall units.

The size of the wire is also important. It is expected that the impact of an excessively long wire is to filter in the spanwise direction, and that this attenuation is proportional to the wire length. According to the estimations in Ligrani and Bradshaw (1987), attenuation becomes an issue with a wire of length beyond 20^+. To generate a valid simulation, the chosen wire size is as big as possible, yet small enough to be realistic. Two different lengths have been simulated: a *small* wire with $l_w = 25^+$, and a *large* one with $l_w = 100^+$. Given that the simulations are run at a constant CFL, and that the time step is proportional to the size of the smallest cell, the simulation with $l_w = 25^+$ did not collect enough data to provide relevant statistics. Unless explicitly noted, all results given correspond to the $l_w = 100^+$. As previously said, the wire is simulated with the sheath, but without the prongs. The aspect ratio of the wire is 100, with a sheath of four times the wire's diameter, and the same length on each side.

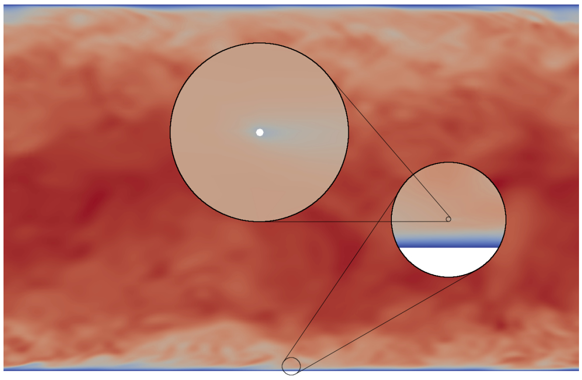

Figure 4: Detail of the thickness of the *small* wire at its slim part, and its relative size compared with the channel. The background image corresponds to an instantaneous cut of the velocity component in the streamwise direction.

Given that this channel flow is periodic in the streamwise direction, this simulation corresponds to a periodic array of wires. The expectation is that, being such a small item compared with the rest of the flow, the feedback of the wake of the wire is not an issue. This condition may change if the wire is simulated with the prongs, that are sensibly bigger than the wire alone.

With this configuration, the relative size of the wire with respect to the complete domain is small, as it is shown in figure 4.

The total drag D is integrated in the thinner measurement zone at each time step, and expressed in wall units with $D^+ = D/\rho\nu^2$. The usual Reynolds decomposition will be used throughout this section: $D = \overline{D} + d$, where the bar superscript stands for the temporal average. This measurement is not strictly equivalent to the one of a hot-wire anemometer, since the temperature distribution is not uniform along the measurement zone, and the heat flux is higher in the central section. On the other hand, total drag is integrated over the wire without any weighting.

The next step has been to compare the fluctuation of the total drag signal d, with the instantaneous fluctuations of the streamwise and wall-normal components of velocity u and v. The following experiment has been carried out. Two different channel flows have been set with the same initial condition, one with the *small* wire, and another one without the wire. In the former, the total drag is computed, and in the latter, the velocity is measured in the central position where the wire would be located. This experiment is valid for a very short time, because the two channels are not expected to have similar outcomes after a long time given the chaotic nature of the flow. The results are presented in figure 5. The signals of d, and u are similar to start with, but after a short time, this similarity is no longer observable, and the total drag of the *cold* wire does not measure the instantaneous velocity at its central point.

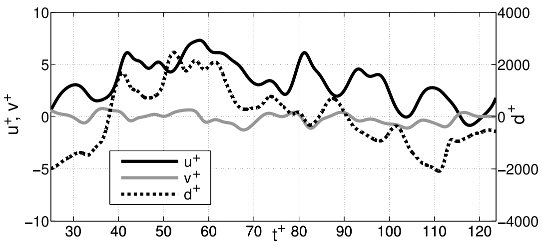

Figure 5: Time evolution of the fluctuations of d (right axis), u and v (left axis) for the *small* wire. The drag is measured over the wire, while velocities are obtained in the wire's central position, repeating the experiment without the wire.

The next test is to determine if the total drag and the streamwise velocity are statistically comparable at the same distance from the wall. A longer simulation has been set up for this purpose. The channel flow with the *large* wire ($l_w = 100^+$), which allows a significantly longer time step, has been let evolve for around $4\pi h/U_\infty$. This is not a long time for a standard turbulent channel flow, particularly with the small domain used in this experiment, but one has to remember that the time step is around ten times smaller due to the presence of the wire. The case with the probe takes 10k CPU hours to run for $\pi h/U_\infty$, while the channel without the anemometer is almost converged with the same amount of resources. The computational resources needed to reach reasonable convergence for most quantities is pretty modest, since 10k CPU hours are equivalent to a middle-sized server running for a month, and memory requirements stay below one gigabyte.

Comparing drag and velocity spectra is not straightforward, and the signal obtained from drag measurements has to be processed. The first step is to obtain the autocorrelation of d:

$$\rho_{dd}(\tau) = \int_{-T}^{T} \frac{d(t)d(t-\tau)}{\sigma(d)^2}\, \mathrm{d}t \tag{1}$$

where σ corresponds to the standard deviation of the signal. The following step is to add the two tails

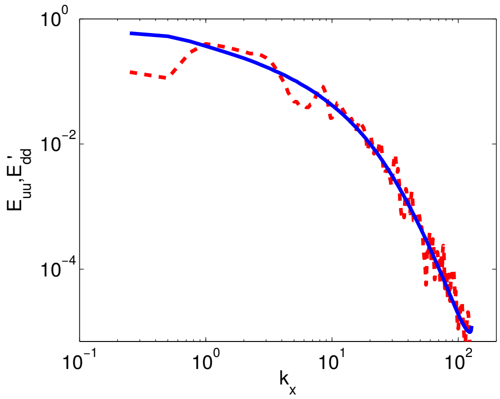

Figure 6: Comparison of streamwise velocity spectrum E_{uu} (blue solid line) at the same distance to the wall, and $Re_\tau = 550$, from del Álamo and Jiménez (2003), and the calibrated drag spectrum E'_{dd} (red dashed line).

to obtain a symmetric correlation, and to window the result, since coherence is not completely lost for long times due to the limited amount of data available. Finally, the processed autocorrelation is Fourier transformed to obtain the spectrum $E_{dd}(\omega)$. The total drag spectrum is obtained as a function of the temporal frequency ω, while the velocity spectrum in del Álamo and Jiménez (2003) is given as a function of the wavenumber in the streamwise direction k_x. According to del Álamo and Jiménez (2009), the measure should be corrected; but since the errors are already important, and the Reynolds number is still small, this correction is not implemented, and the usual Taylor's approximation is applied. Since the two spectra don't contain the same energy, the drag spectrum has been rescaled to to match the velocity fluctuations using the following formula:

$$E'_{dd}(k_x) = \frac{E_{dd}(k_x) \int E_{uu}(k_x)\, \mathrm{d}k_x}{\int E_{dd}(k_x)\, \mathrm{d}k_x} \tag{2}$$

The comparison between E'_{dd} and E_{uu} is presented in figure 6.

As expected, there is not sufficient data to obtain a converged spectrum and E'_{dd} is quite noisy. But it is important to remember that this result tests the correctness of the simulation to the extreme, and it ignores more than 99% of the data generated by the simulation.

Another interesting result that can be extracted from this simulation, which is the flow field around the *cold* wire, is presented in figure 7. The perturbation introduced by the wire resembles the one obtained in the simplified case with a similar Re_D, both in shape and in size. The laminar wake is several times longer than the wire's diameter, but small compared to the distance from the wall, and does not seem to be affected by the presence of the wire. On the other hand, the fact that the perturbation is several times bigger than the diameter may be important in the case of probes with cross wires. The length of the perturbation is around ten times larger than the probe, but since the smallest energy-containing streamwise wavelengths are around a hundred times larger than the diameter of the wire (see Smith and Metzler (1983)), and ten times larger than the perturbation, that does not imply that the spectrum of the streamwise component of velocity is modified by the presence of the wire.

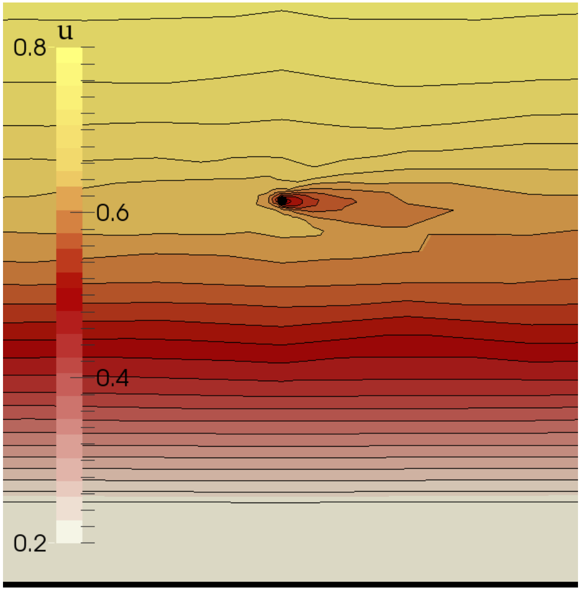

Figure 7: Average streamwise component of velocity in the central section of the *large* wire, perpendicular to its axis. Velocity is given in arbitrary units. The average velocity in the channel centerline is $U_{cl} = 1.17$ velocity units. The thick black line at the bottom of the figure is the wall.

5 Conclusions and future work

The most important conclusion of this work, considering that this project is at a preliminary stage, is that it is possible to simulate a hot-wire anemometer in a realistic environment, and it can be used to understand constant temperature anemometry's limitations. The usefulness of this setup to analyze the behavior of the probe itself is arguable, but the complete description of the flow field around the wire can be obtained. The presented simulation is relatively inexpensive, and can be easily extended to analyze more exotic probes. This result also confirms that OpenFOAM is a valid tool for DNS of non-trivial configurations, and that good results can be obtained if used properly. A channel flow has been set up, and run until convergence, with a moderate amount of resources; and the configuration of the case was simple enough to be a model project.

There are other open questions about the correctness of the present results, which have not been analyzed in sufficient detail, but the first impression looks promising.

The complete description of the flow field around the hot-wire anemometer, which is still in a preliminary state, is the most promising outcome of this work. It seems clear that the perturbation caused by the probe is unlikely to affect the length energy-containing eddies that are close to the wall. On the other hand, since the width of those eddies is comparable to the length of the wire, the perturbation caused by the probe is expected to have a relevant effect. The study of this phenomenon requires a three-dimensional description of the flow around the wire, that is not available yet.

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